Solution: Bound and Scattering States of
One Dimensional Quantum Mechanical Systems

Warm-up:

1.

(I) *Correct. The car is in a classical bound state.*
(II) *Correct. The car is bound between x=a and x=b. It cannot be found outside that region, the car is classical.*
(III) *Correct. x=a and x=b are called turning points, and since the car cannot be found beyond those points, it does turn back when it reaches those points.*

(a) No. Statement (I) is correct but there are other ones that are correct.
(b) No. Statements (I) and (II) are correct but there are other ones that are correct.
(c) No. Statements (I) and (III) are correct but there are other ones that are correct.
(d) No. Statements (II) and (III) are correct but there are other ones that are correct.
(e) *Correct. All of the above statements are correct statements..*

2.

(I) *The electron is in a quantum mechanical bound state.*
(II) This is a quantum mechanical system, the electron can be found outside the classically allowed region. Its wave function however, needs to be exponentially decaying in the classically forbidden region.
(III) *Since this is a quantum mechanical system, the electron can be found outside the classically allowed region.*

(a) No. (I) is correct but there are other ones that are correct.
(b) No. The electron can be found outside the classically allowed region.
(c) No. Statement (III) is correct but there are other ones that are correct.
(d) No. One of the statements (I) and (II) is incorrect.
(e) *Correct. (I) and (III) are correct statements.*

3.
(a) *Correct. It is in a bound state between x=a and x=b because the potential is higher than the total energy.*
(b) No. It cannot be found outside the region between x=a and x=b, because its total energy is less than the potential energy at the boundaries of that region.
(c) No. The mass and the speed are not necessary information to determine whether this is a bound or a scattering state.
(d) No. The exact position between x=a and x=b need not be known, just knowing it is between x=a and x=b is enough.
(e) No. One can have a potential energy of any reasonable shape, this is as reasonable as any other.

4.

(a) No. The electron is in a bound state but in quantum mechanics it does not mean that it is stuck between x=a and x=b.

(b) *Correct. The electron is in a bound state, and there is a finite probability that it will be found outside the classically allowed region. The wave function needs to be exponentially decaying in that region, and cannot be found at either infinity.*

(c) No. The fact that there is a probability of the electron being found outside does not mean it is in a scattering state.

(d) No. The initial state has nothing to do with this.

(e) No. This is a valid potential energy, it is as reasonable as any other.

5.

(a) No. The electron is not in a bound state.

(b) No. The electron can be found in the region between x=a and x=b. Even if it wasn't if an electron can be found at either infinity it is not bound.

(c) *Correct. This is a scattering state, because the electron can be found at infinity.*

(d) No. The initial position does not determine whether this is a bound or a scattering state.

(e) No. The Hamiltonian operator can be determined from the given information, we know the total energy and the potential energy, and that is sufficient.

6.

(I) No. It has less energy than the potential between $x=a$ and $x=b$ but there is a finite probability that it will tunnel to the other side.

(II) *Correct. The electron can tunnel to the region where its energy is less than the potential energy, provided the potential energy is finite.*

(III) *Correct. The electron can be found at* $x>b$.

(a) No. Statement (I) is incorrect.

(b) No. Statement (II) is correct but there are others that are also correct.

(c) No. Statement (III) is correct but there are others that are also correct.

(d) No. One of the statements (I) and (II) is incorrect.

(e) *Correct. (II) and (III) are the only two correct statements.*

7.

(I) No. Bound states in quantum mechanics and classical mechanics are not the same.

(II) No. In quantum mechanics, a bound particle can tunnel into the classically forbidden region, but it has a decaying wave function. It cannot be found at infinity.

(III) *Correct. The particle can tunnel into the classically forbidden region, but its wave function will decay.*

(IV) No. The particle cannot have a sinusoidal wave function in the classically forbidden region, it has to have a decaying wave function.

(a) No. Statement (II) is incorrect.
(b) *Correct. Statement (III) is the only correct statement.*
(c) No. Statement (IV) is not correct, the wave function cannot be sinusoidal in the classically forbidden region.
(d) No. Both statements (I) and (II) are incorrect. Classical and quantum mechanical bound states are different, and in quantum mechanics the particle can be found in the classically forbidden region with a finite probability.
(e) No. One of the statements (III) and (IV) is incorrect.

8.

(I) *Correct. The wave function of a particle in a bound state must decay to zero at both infinities.*

(II) *Correct. If the total energy of the particle is less than the potential energy at both plus and minus infinity, the particle is in a bound state.*

(III) No. The energy has to be smaller than the potential energy at both infinities .If the particle can be found at either infinity, it is not bound.

(a) No. Statement (I) is correct but there are others that are also correct.
(b) No. Statement (II) is correct but there are others that are also correct.
(c) No. Statement (III) is incorrect.
(d) *Correct. Both statements (I) and (II) are correct and there is not another statement that is correct.*
(e) No. One of the statements (II) and (III) is incorrect.

9.

(I) *Correct. The electron does feel an attractive force due to the presence of the proton.*

(II) *Correct. Since the electron is bound in the Hydrogen atom, it does have a discrete energy spectrum.*

(III) No. It does not have a continuous energy spectrum. Energy levels are quantized for bound particles.

(a) No. Statement (I) is correct but there are others that are also correct.
(b) No. Statement (II) is correct but there are others that are also correct.
(c) No. The electron does not have a continuous energy spectrum, it can only have certain energies.
(d) *Correct. (I) and (II) are the only correct choices.*
(e) No. One of the statements (II) and (III) is incorrect.

10.

(I) *Correct. An unbound electron will also feel the attractive force due to the presence of the proton when it is in the vicinity of the proton.*

(II) No. The energy spectrum for an electron in a scattering state is continuous not discrete.

(III) *Correct. All energies are allowed for an electron in a scattering state in the vicinity of a proton.*

(a) No. (I) is correct but there are others that are correct.
(b) No. The energy spectrum is not discrete, it is continuous.
(c) No. One of the statements (I) and (II) is incorrect.
(d) *Correct. (I) and (II) are the only correct statements.*
(e) No. One of the statements (II) And (III) is incorrect.

11.

(I) *Correct. Bound states have energy* $-13.6eV/n^2$ *where n is a positive integer.*

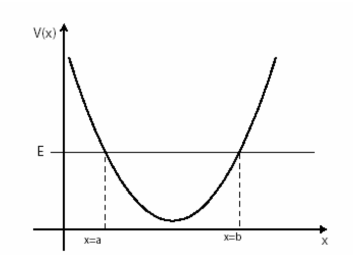
(II) *Correct. An electron with energy greater than zero will be in a scattering state. (Note that the allowed energies for a bound electron are all negative).*

(III) No. Scattering states have a continuous energy spectrum.

(a) No. Statement (I) is correct but there are others that are also correct.
(b) No Statement (II) is correct but there are others that are also correct.
(c) No. Scattering states do not have a discrete energy spectrum.
(d) *Correct. Statements (I) and (II) are the only correct choices.*
(e) No. One of the statements (II) and (III) is incorrect.

Pretest

1. A particle in a classical bound state cannot be found outside the classically forbidden region. A quantum mechanical bound state means the particle can be found outside the classically forbidden region with a finite probability, but it cannot be found at either infinity. The probability of finding it in the classically forbidden region is rapidly decaying.
2.



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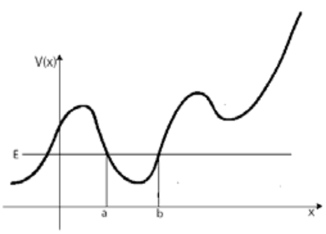
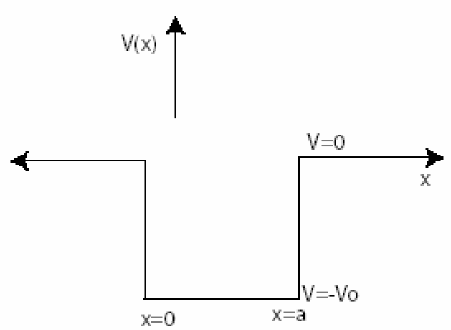


This potential energy allows for only scattering states because no matter what the total energy of the particle is, it will be in a scattering state. Notice that energies smaller than zero are not allowed at all.

4. A very typical example is the finite square well. There are other examples.



5. A particle will be in a bound state if the potential energy at either infinity is greater than the total energy of the particle. In other words, if the particle cannot be found at infinity, it is bound. Unlike the classical case, the particle can be found outside the classically forbidden region, but it cannot be found ar infinity. A particle will be in a scattering state if the total energy of the particle is greater than the potential energy at either or both infinities. If a particle can eb found at infinity, it is in a scattering state.

Tutorial

1.

(a) *Correct. The particle is in a bound state because it energy at $x = \pm\infty$ is less than the potential energy.*

(b) No. The particle cannot be found at either infinity, it is not in a scattering state.

(c) No. A particle is either in a scattering state or a bound state, it cannot be in one at a position and in the other at another position.

(d) No. Such an energy level is possible for a particle. The energy need not look like the potential. It can be greater than the potential at some points and smaller than the potential energy at other points.

(e) No. The wave function is not necessary to determine whether an electron with the given energy will be in a bound or a scattering state in the given potential well.

2.

(a) No. The energy of the electron is larger than the potential energy at both infinities. This particle is not bound.

(b) *Correct. The particle is in a scattering state because its total energy is higher than the potential energy at both infinities.*

(c) No. The energy need not look like the potential well, it can be higher at some points and lower at others than the potential energy.

(d) No. The wave function is not necessary to determine whether an electron with the given energy will be in a bound or a scattering state in the given potential well.

(e) No. There is a correct answer choice above.

3.

(I) No. The particle will feel the presence of a force. It will not keep moving unaffected by the potential well.

(II) *Correct. The particle can bounce back and be found at $x = -\infty$.*

(III) No. Inside the well is classically allowed to the particle, it will not have a decaying wave function inside the well.

(a) No. The particle will not move in a straight line with 100% certainty.

(b) *Correct. (II) is the only correct statement.*

(c) No. The wave function will not decay inside the well. That region is classically allowed.

(d) No. One of the statements (I) and (II) is incorrect.

(e) No. There is a correct answer choice above.

4.

(a) No. The finite square well allows for both bound and scattering states.
(b) No. The finite square well allows for both bound and scattering states.
(c) No. A particle is either in a bound state or a scattering state. In cannot change from one to the other with position.
(d) No. It can be in a scattering state or a bound state but the position does not determine that, the particles energy compared to the potential energy does.
(e) *Correct. A particle with energy less than zero will be in a bound state and a particle with energy more than zero will be in a scattering state, in this finite square well.*

5.

(I) No. The particle in a bound state in the finite square well can tunnel to the classically forbidden region because the energy outside the well is finite.
(II) *Correct. Although the particle can tunnel into the classically forbidden region, its wave function in that region will decay exponentially. The wave function will be zero at infinity, so the particle cannot be found at infinity.*
(III) No. The wave function can be more localized than the well, as long as it is continuous, normalizable and smooth.

(a) *Correct. Statement (II) is the only correct statement.*
(b) No. One of the statements (I) and (II) is incorrect.
(c) No. The particle can tunnel to the classically forbidden region, but it doesn't have to. Its wave function can also be more localized than the well.
(d) No. One of the statements (II) and (III) is incorrect.
(e) No. Some of the statements are incorrect.

6.

(I) *Correct. The wave function decays exponentially in the classically forbidden region.*
(II) *Correct. Bound state wave functions are normalizable because the go to zero at infinities.*
(III) *Correct. Particles in bound states cannot be found at infinity.*

(a) No. (II) is correct but there are other correct statements.
(b) No. (I) and (II) are correct but there are other correct statements.
(c) No. (I) and (III) are correct but there are other correct statements.
(d) No. (II) and (III) are correct but there are other correct statements.
(e) *Correct. All of the above statements are correct.*

7.

(I) No. Its wave function can be nonzero outside the well, there is a finite probability that the particle will be found outside the well.
(II) *Correct. A particle with positive energy will be in a scattering state, for this finite square well.*
(III) *Correct. A particle in a bound state can only have discrete energies.*

(a) No. The wave function can be nonzero outside the well.
(b) No. (II) is correct, but there are other statements that are also correct.
(c) No. One of the statements (I) and (II) are incorrect.
(d) No. One of the statements (I) and (III) are incorrect.
(e) *Correct. (II) and (III) are the only correct statements.*

8.

(I) No. A particle is either in a bound state or a scattering state. In cannot change from on to the other with position.
(II) *Correct. The particle is in a bound state because its energy is less than the potential energy at both infinities. Its wave function will be exponentially decaying outside the well because the particle is classically forbidden to be there.*
(III) No. The finite square well does allow bound state solutions, and this particle is in a bound state.

(a) No. The particle cannot change from a bound state to a scattering state or vice versa with changing position.
(b) *Correct. Statement (II) is the only correct statement.*
(c) No. The finite square well allows for bound states and this electron is in a bound state.
(d) No Both (I) and (III) are incorrect. The particle cannot change from being in a bound state to a scattering state with the same energy. The finite square well does allow for bound states.
(e) No. One of the choices above is correct.

9.

(I) *Correct. The electron will be attracted to the potential energy well regardless of whether it is in a bound state or a scattering state.*
(II) No. The energy levels are only discrete for bound states.
(III) No. The wave function is only normalizable for bound states.

(a) *Correct. (I) is the only correct statement.*
(b) No. Energy levels are continuous for scattering states.
(c) No. Scattering state wave functions are not normalizable.
(d) No. One of the statements (I) and (II) is incorrect.
(e) No. there is a correct answer above.

10. *Reyna's reasoning is correct.* Scattering states are not normalizable, but one can obtain normalizable wave packets from a linear superposition of scattering states. Your drawing should be a wave packet also.

11.

(a) No. The particle is not in a bound state because it can be found at infinity.
(b) *Correct. The electron is in a scattering state because its total energy is higher than the potential energy at infinity. The particle can be found at infinity.*
(c) No. This energy is possible. The total energy of the particle need not reflect the symmetry of the barrier.
(d) No. Knowing the wave function is not necessary to determine whether the electron with the given energy will be in a bound or scattering state in the given potential energy barrier.
(e) No. There is a correct answer among the choices above.

12.

(a) No. The particle is not in a bound state because it can be found at infinity.
(b) *Correct. The electron is in a scattering state because its total energy is higher than the potential energy at infinity. The particle can be found at infinity.*
(c) No. This energy is possible. The total energy of the particle need not reflect the symmetry of the potential energy barrier.
(d) No. Knowing the wave function is not necessary to determine whether the electron with the given energy will be in a bound or scattering statre in the given potential energy barrier.
(e) No. There is a correct answer among the choices above.

13.

(a) No. The barrier does not allow for bound states at all.
(b) *Correct. The barrier only allows for scattering states.*
(c) No. It only allows for one of those, not both.
(d) No. This is a possible energy in quantum mechanics.
(e) No. The Hamiltonian operator need not be known. We know the potential energy, and that is sufficient in determining what states will be allowed.

14. *Barney is correct.* The barrier is a repulsive potential, and particles that are repelled by each other will not be bound together. That is why the barrier does not allow for bound states.

15.

(I) *Correct. The infinite square well only allows for bound states because the particle cannot be found at infinity.*

(II) No. In general the wave function of a particle does decay exponentially in the classically forbidden region. But since the potential outside the well is infinite, the wave function will go to zero abruptly at the boundaries. The probability of finding the particle outside the well is exactly zero.

(III) *Correct. Since the potential outside the well is infinite, the wave function will go to zero, making the first derivative discontinuous.*

(a) No. This statement is correct, but there are others that are also correct.
(b) No. The wave function does not decay exponentially outside the well, it goes to zero abruptly.
(c) No. (III) is correct but there are other statements that are also correct.
(d) No. One of the statements (I) and (II) is incorrect.
(e) *Correct. (I) and (III) are the only two correct statements.*

16.

(I) *Correct. The potential energy is infinite at infinity, therefore only bound states are allowed. There cannot be a particle that has higher energy than the potential energy at infinity.*

(II) *Correct. Since the potential energy does not become infinite immediately, there is a finite probability that the particle will tunnel into the classically forbidden region. The particle cannot be found at infinity, so its wave function will be exponentially decaying in the classically forbidden region.*

(III) No. The wave function will not go to zero abruptly because the potential energy does not become infinite before $x \to \pm\infty$.

(a) No. (I) is correct but there are other statements that are also correct.
(b) No. (II) is correct but there are other statements that are also correct.
(c) No, the wave function will not go to zero immediately.
(d) *Correct. (I) and (II) are the only correct statements.*
(e) No. One of the statements (I) and (III) are incorrect.

17.

(I) No. The infinite square well only allows for bound states.

(II) *Correct. The finite square well allows for both bound and scattering states.*

(III) No. The harmonic oscillator only allows for bound states.

(a) No. Infinite square well does not allow both bound and scattering states.
(b) *Correct. Finite square well allows for both bound and scattering states, and there are no others among the above three that do.*
(c) No. The SHO does not allow for both bound and scattering states.
(d) No. One of those potentials do not allow for both bound and scattering states.
(e) No. One of those potentials do not allow for both bound and scattering states.

18.

(I) No. The wave function does not decay inside the finite square well for any energy.

(II) *Correct. The scattering state wave function must be nonzero at infinity. Inside the well is also allowed, so the wave function there must also be sinusoidal.*

(III) No. The wave function is not allowed because it is not continuous. Even if it were, for a particle in a scattering state, the probability of finding it at infinity should be nonzero.

(a) No. That is not a scattering state wave function for the finite square well.
(b) *Correct. (II) is the only correct figure for a scattering state wave function.*
(c) No. That wave function is not allowed at all, for bound or scattering states, because it is not continuous to begin with.
(d) No. One of the figures (I) and (II) is incorrect.
(e) No. One of the figures (II) and (III) is incorrect.

19.
(a) No. In the hydrogen atom, the electrons can be found outside the well. They can be in scattering states.
(b) *Correct. The probability of leaking outside the chain is zero, which means the particles encounter an infinite repulsive potential barrier at the ends of the chain. The infinite square well is the best description.*
(c) No. The simple harmonic oscillator also only allows for bound states, but the probability of finding the particle outside is not zero.
(d) No. The delta function potential well has width of unity, where as here there is a finite dimensional chain.
(e) No. There is a correct answer among the choices above.

20.

(I) *Correct. The depth of the well is the repulsive potential barrier the electrons in the atom face, which is a property of the type of metal used.*

(II) *Correct. The width of the well is the physical width of the chain.*

(III) *Correct. The energy levels depend on the effective mass and the width of the well. Since the effective mass is the same, as the width increases, the energy levels get closer together.*

(a) No. (I) is correct but there are other correct statements.
(b) No. (II) is correct but there are other correct statements.
(c) No. (I) and (II) are correct, but there are other correct statements.
(d) No. (II) and (III) are correct but there are other correct statements.
(e) *Correct. All of the above statements are right.*

21.

(a) *Correct. Since the chain is infinitely long, the particles can be found at any point between* $x = -\infty$ and $x = +\infty$. *The best model is the free particle. The fact that the potential energy is not zero in the chain does not change this, since it is the same everywhere in the chain.*
(b) No. This is an infinitely wide one dimensional chain. The delta function potential well is an infinitely deep one dimensional potential.
(c) No. The finite square well has finite width. The potential is different outside the well than it is inside. This model is different; it has the same potential from infinity to infinity.
(d) No. The simple harmonic oscillator is also a changing potential, not a good model to describe an infinitely long constant potential.
(e) No. There is a correct answer among the choices above.

22.

(a) No. The electrons have a finite probability of being found in the A type atom region.
(b) *Correct. The electrons feel a finite repulsive potential in the A region, but there is a finite probability of leaking to the A region.*
(c) No. The simple harmonic oscillator potential changes with position. It is not a two step potential like the finite square well.
(d) No. The delta function well is infinitely narrow, whereas the chain of type B atoms has finite width.
(e) No. There is a correct answer among the choices above.

Posttest

1. The bound state energy should be drawn between $V=0$ and $V=-V_0$. That is, $-V_0 < E_b < 0$. The scattering state energy should be drawn above $V=0$ level. That is $E_s > 0$.

2. No. Such an energy is not possible. An electron is either in a bound state or a scattering state. The energy of the electron is fixed.

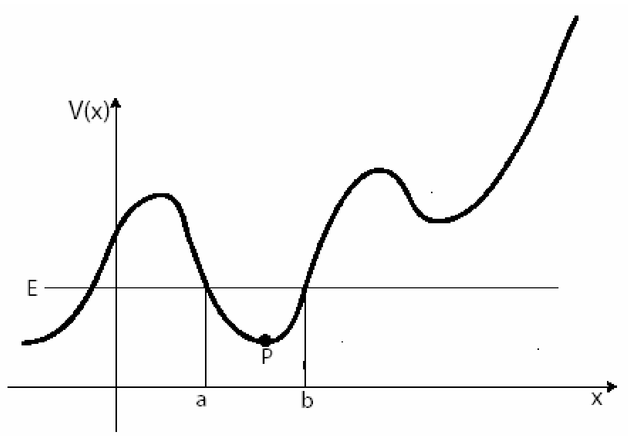
3.

(a) It is in a bound state. The energy of the electron at either infinity is less than the potential energy, so it is in a bound state.

(b) Yes there is. It is in a bound state but the probability of finding it outside the region between $x=-a$ and $x=a$ is non zero. The potential energy does go to infinity, but it does so gradually, not abruptly.

(c) No. The electron cannot be found at infinity. It is in a bound state. The potential energy at infinity is infinite, the particle cannot be found there.

4.



A classical particle initially at point P will be in a bound state. An electron initially localized around P will be in a scattering state because it can be found at $x=-\infty$. There is a probability that the particle will tunnel through the potential energy to its left even though the potential energy is higher.

There are other examples. Any potential energy diagram with a well higher than the particle energy will be a classical bound state. If the potential energy is finite at either $x=\pm\infty$, it will be quantum mechanically a scattering state.

5. A potential energy diagram that satisfies the given conditions cannot be drawn, it is impossible. There is no potential energy that will allow for quantum mechanically bound but classically scattering states. Quantum mechanically bound states are possible when the potential energy at both $x=\pm\infty$ is infinite, in which case a classical scattering state is not allowed.